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Erling Jensen

GENERAL THEORY ON SPIN-ECHOES FOR ANY COMBINATION OF ANY NUMBER OF PULSES

Introduction of a Simple "Spin-Echo Diagram"

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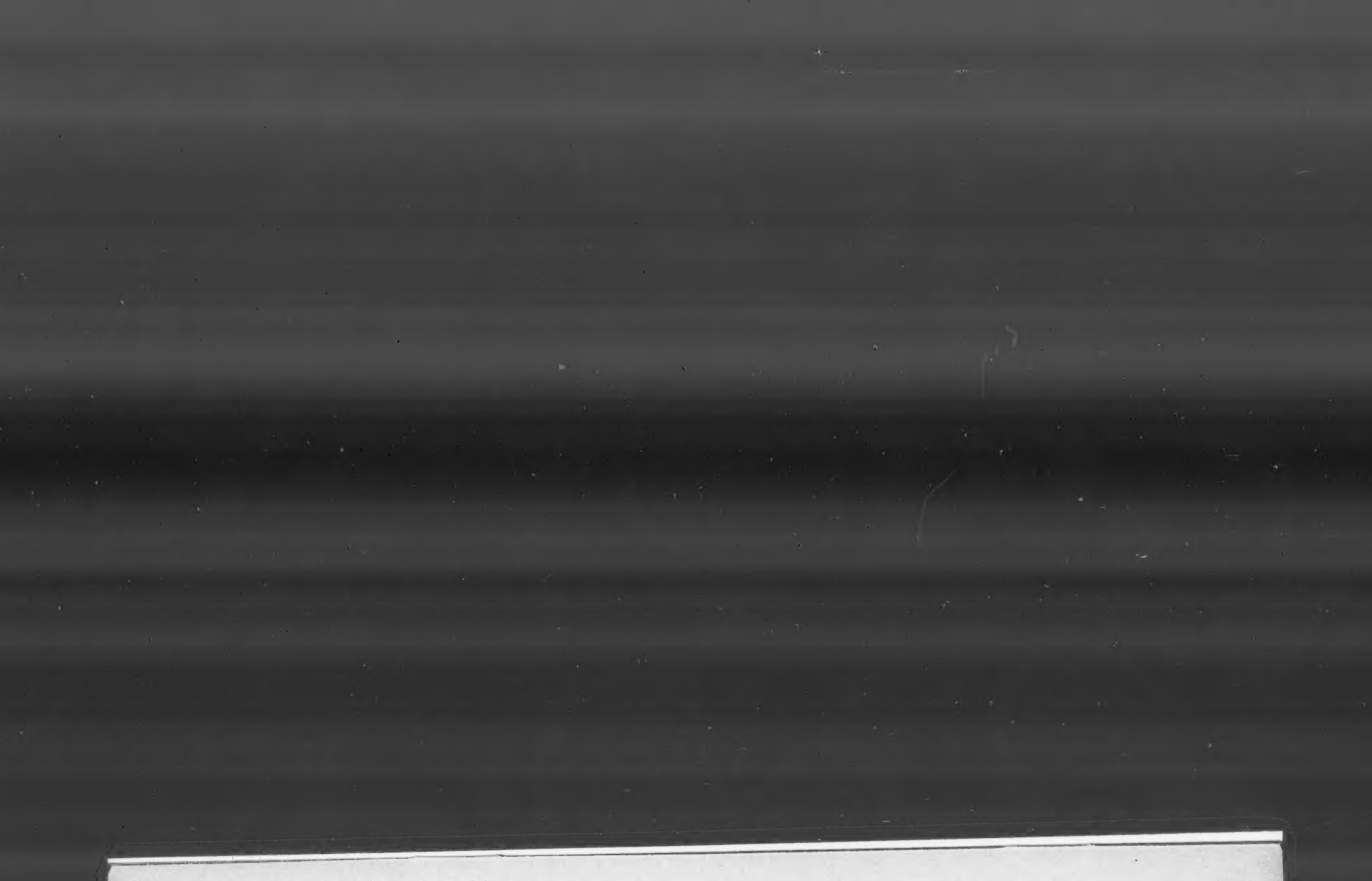

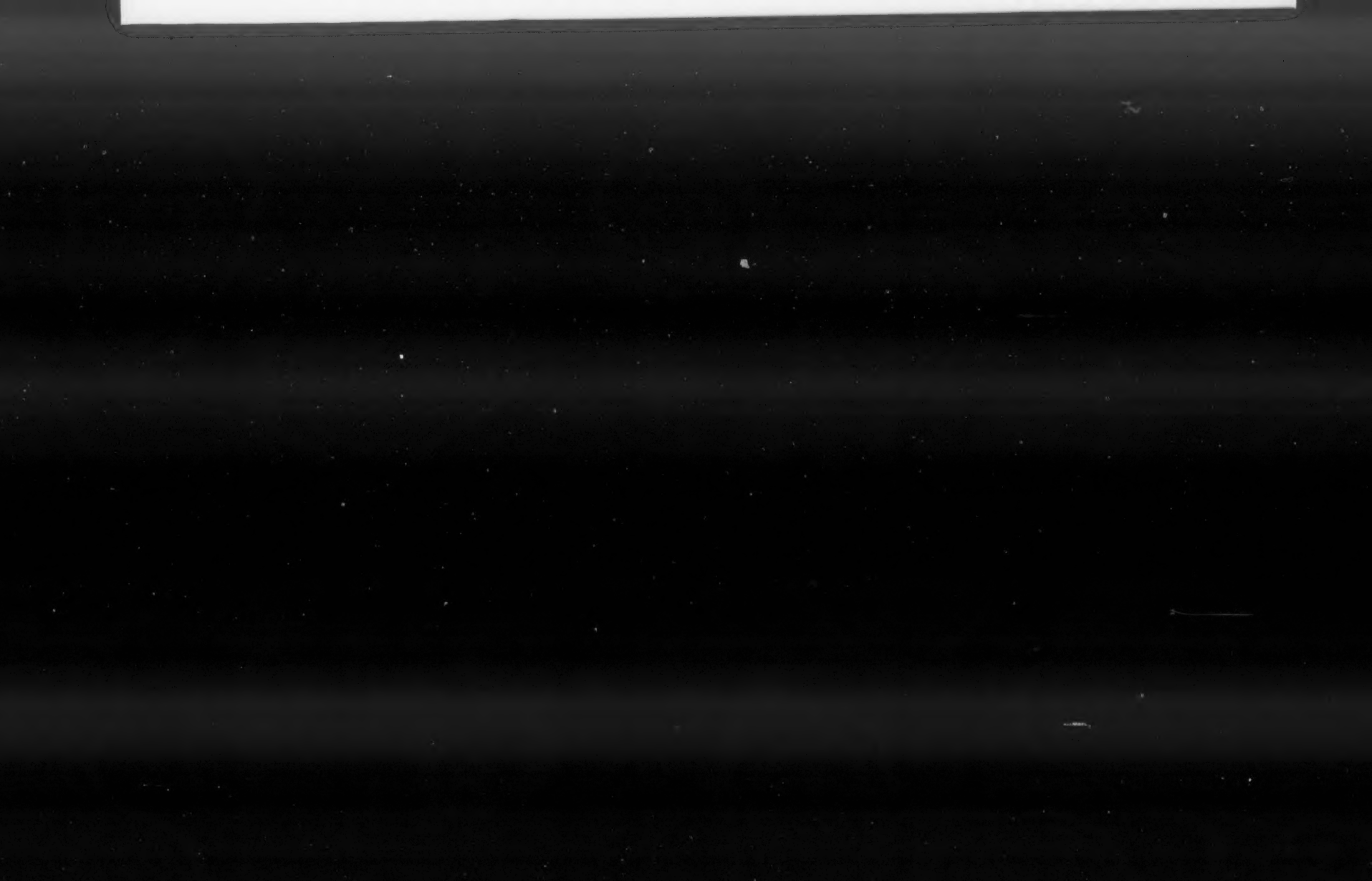

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Erling Jensen
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SUMMARY

The effects of diffusion damping of spin-echo amplitudes were first treated by Hahn[1]. An improvement of Hahn's averaging procedure was suggested by Das and Saha[2]. Their theory, however, as is shown here, contains an error. This is corrected, and the revised theory is shown to give for a special case the same result as obtained by different methods by Carr and Purcell[3] and by Torrey[4]. Furthermore the revised theory is extended to account for any combination of pulses, and it is shown that the results are conveniently summarized in a simple "spin-echo diagram".

GENERAL THEORY ON SPIN-ECHOES FOR ANY COMBINATION OF ANY NUMBER OF PULSES. INTRODUCTION OF A SIMPLE "SPIN-ECHO DIAGRAM"

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Introduction

The damping effects of diffusion on spin-echo amplitudes were treated in Hahn's original work[1]. However, an improvement in Hahn's averaging procedure was suggested by Das and Saha[2] who pointed out that the damping due to diffusion after a single pulse might be affected by successive pulses. The special case of a 90° pulse followed by a sequence of 180° pulses was treated by Carr and Purcell[3] who used a simple and convincing random-walk theory. Application of the theory of Das and Saha on the Carr-Purcell pulse method, however, yields a result which is not in agreement with that obtained by Carr and Purcell. As will be pointed out here the source of this discrepancy is that in the theory of Das and Saha two sets of statistical variables are treated as if they were mutually independent when, in fact, they are directly correlated.

Here will be given a theory, which takes account of this correlation and may be characterized as a combination of the theories of Das and Saha and of Carr and Purcell. It gives as a special case the result of Carr and Purcell but furthermore is used to derive a general solution for all the spin-echoes due to any combination of pulses. The results are most conveniently summarized in a simple "spin-echo diagram" that will be introduced later on.

Summary of previous theories and critical discussion of same

For later use it will be convenient to notice the following relations and to summarize some previous results. Let

$$X = \sum_{i=1}^{n} X_i \tag{1}$$

where the X_i are mutually independent statistical variables with even density functions $p(X_i)$. Then

$$E(\cos X) = \int_{-\infty}^{\infty} \cos X \; p(X)\, dX = \prod_{i=1}^{n} E(\cos X_i) = \prod_{i=1}^{n} \int_{-\infty}^{\infty} \cos X_i \; p(X_i)\, dX_i \tag{2}$$

as all other terms contain at least one independent factor $\sin X_i$, and thus integrate to zero. If furthermore the X_i are normal random variables

$$E(\cos X) = \exp\left[-\tfrac{1}{2} E(X^2)\right] = \exp\left[-\tfrac{1}{2} \sum_{i=1}^{n} E(X_i^2)\right] . \tag{3}$$

Only taking account of self-diffusion with the self-diffusion coefficient D and assuming a constant field gradient $\overline{G}$, $\left(G = \frac{dH_z}{dl}\right)$

$$\eta_{t,\, t_1} = \gamma G l_{t,\, t_1} \tag{4}$$

$$E(\eta_{t,\, t_1})^2 = \gamma^2 G^2 E(l_{t,\, t_1})^2 = 2\gamma^2 G^2 D(t - t_1) = 2k(t - t_1) \tag{5}$$

where γ is the gyromagnetic ratio, $l_{t,\, t_1}$ is the diffusion length in the time interval $t - t_1$, and η_{t,t_1} is the corresponding random increase in angular velocity.

Then the random phase accumulation at time t after a single pulse supplied at time 0 is (see ref. 3)

$$\phi_{t,\, o} = \int_o^t \eta_{t;\, o}\, dt' = \lim_{N \to \infty} \frac{t}{N} \sum_{j=1}^{N} \sum_{i=1}^{j} \eta_i = \qquad , \qquad \left(\eta_i = \eta_{(i-1)\frac{t}{N},\; i\frac{t}{N}}\right) .$$

$$\lim_{N\to\infty} \frac{t}{N} \cdot \begin{bmatrix} +\eta_1 \\ +\eta_1 + \eta_2 \\ +\eta_1 + \eta_2 + \eta_3 \\ +\ldots+\ldots+\ldots+\ldots \\ +\eta_1 + \eta_2 + \ldots + \ldots + \eta_N \end{bmatrix} = \lim_{N\to\infty} \frac{t}{N} \sum_{j=1}^{N} (N+1-j)\eta_j \qquad (6)$$

Since the η_j are mutually independent statistical variables with mean value zero, and by using (5)

$$E(\phi_{t,o})^2 = \lim_{N\to\infty} (\frac{t}{N})^2 \sum_{j=1}^{N} j^2 E(\eta_j)^2 = (\frac{t}{N})^2 \frac{N^3}{3} \frac{2kt}{N} = \frac{2}{3}kt^3 \qquad (7)$$

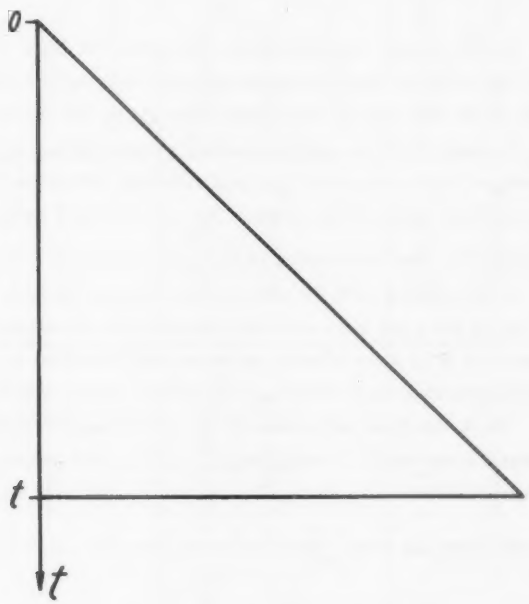

Fig. 1. Representation for $\phi_{t,o}$.

For simplicity the array (6) for $\phi_{t,o}$ will merely be symbolized by the triangle fig. 1 containing the η_i and characterized by the indicated time axis.

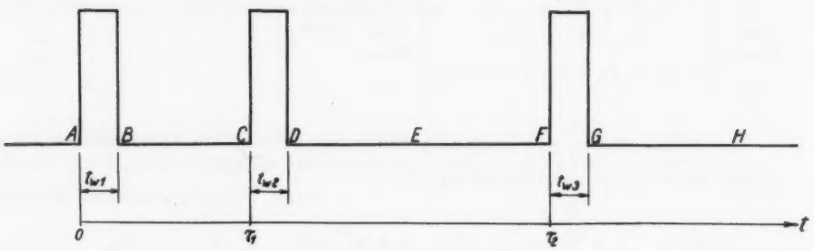

Fig. 2. Applied pulses at the radio-frequency.

The pulse array under consideration is shown in fig. 2, and in table I (obtained by slightly modifying table I, ref. 2, pag.751, which is derived directly from the Bloch equations) are given the normalized components U, V, and W of the magnetization vector $\overline{M}(\Delta\omega)$ when the values are averaged with respect to $\Delta\omega$ and suitable diffusion variables. To express the random part of the phases A_{t_2, t_1} Das and Saha make use of both ϕ's and η's (the first form at the bottom of table I), whereas it is preferred in this paper only to use ϕ's (the second form).

Assuming $\Delta\omega$ to have an even density function the U-component integrates to zero as it is seen always to be an odd function of $\Delta\omega$, which is of course independent of any diffusion variable. The signal therefore is simply V(t). Thus the free induction decay following the first pulse, $0 < t < \tau_1$, is found from table I replacing τ_1 with t, and according to the above characteristics of $\Delta\omega$ using (2), and thereupon (3) and (7),

$$-\sin\xi_1 \exp(-\frac{t}{T_2}) E\left[\cos(\Delta\omega t + \phi_{t,o})\right] =$$

$$-\sin\xi_1 \exp(-\frac{t}{T_2}) E\ (\cos\Delta\omega t) E(\cos\phi_{t,o}) =$$

$$-\sin\xi_1 \exp(-\frac{t}{T_2}) E\ (\cos\Delta\omega t) \exp(-\frac{1}{3}kt^3) \qquad (8)$$

Table I. Values of U, V, and W at instants A, B, C, D, F, and H, to be averaged.

Instant	0(A)	t_{w1}(B)	τ_1(C)	$\tau_1 + t_{w2}$(D)	τ_2 (F)	$\tau_2 + t_{w3}$(G)	$t > \tau_2 + t_{w3}$(H)
U	0	0	$\sin\xi_1 \sin A_{\tau_1, o} \times \exp(-\frac{\tau_1}{T_2})$	U(C)	$\left[U(D)\cos A_{\tau_2, \tau_1} - V(D)\sin A_{\tau_2, \tau_1}\right] \times \exp(-\frac{\tau_2 - \tau_1}{T_2})$	U(F)	$\left[U(G)\cos A_{t, \tau_2} - V(G)\sin A_{t, \tau_2}\right] \times \exp(-\frac{t - \tau_2}{T_2})$
V	0	$-\sin\xi_1$	$-\sin\xi_1 \cos A_{\tau_1, o} \times \exp(-\frac{\tau_1}{T_2})$	$V(V)\cos\xi_2 - W(C)\sin\xi_2$	$\left[U(D)\sin A_{\tau_2, \tau_1} + V(D)\cos A_{\tau_2, \tau_1}\right] \times \exp(-\frac{\tau_2 - \tau_1}{T_2})$	$V(F)\cos\xi_3 - W(F)\sin\xi_3$	$\left[U(G)\sin A_{t, \tau_2} + V(G)\cos A_{t, \tau_2}\right] \times \exp(-\frac{t - \tau_2}{T_2})$
W	1	$\cos\xi_1$	$1 - (1 - \cos\xi_1) \times \exp(-\frac{\tau_1}{T_1})$	$V(C)\sin\xi_2 + W(C)\cos\xi_2$	$1 - \left[1 - W(D)\right] \times \exp(-\frac{\tau_2 - \tau_1}{T_1})$	$V(F)\sin\xi_3 + W(F)\cos\xi_3$	$1 - \left[1 - W(G)\right] \times \exp(-\frac{t - \tau_2}{T_1})$

$$A_{\tau_1, o} = \Delta\omega\tau_1 + \phi_{\tau_1, o}$$

$$A_{\tau_2, \tau_1} = (\Delta\omega + \eta_{\tau_1, o})(\tau_2 - \tau_1) + \phi_{\tau_2, \tau_1} = \Delta\omega(\tau_2 - \tau_1) + (\phi_{\tau_2, o} - \phi_{\tau_1, o})$$

$$A_{t, \tau_2} = (\Delta\omega + \eta_{\tau_1, o} + \eta_{\tau_2, \tau_1})(t - \tau_2) + \phi_{t, \tau_2} = \Delta\omega(t - \tau_2) + (\phi_{t, o} - \phi_{\tau_2, o})$$

$$\xi_n = \omega_1 \cdot t_{wn}$$

Assuming here for simplicity $\Delta\omega$ to be normally distributed with variance $(\frac{1}{T_3})^2$, (3) gives directly

$$-\sin\xi_1 \exp(-\frac{t}{T_2})\exp(-\frac{t^2}{2T_3^2})\exp(-\frac{1}{3}kt^3) \tag{9}$$

For the primary echo Hahn now simply uses the diffusion factor for the decay following a single pulse, inserting $t = 2\tau_1$

$$E(\cos\phi_{2\tau_1,\,o}) = \exp(-\frac{8}{3}k\tau_1^3) \tag{10}$$

Das and Saha, however, assuming $\phi_{\tau_1,\,o}$, $\phi_{2\tau_1,\tau_1}$ and $\eta_{\tau_1,\,o}$ to be mutually independent, even normally distributed statistical variables, obtain for the primary echo, and for the (by the second pulse modified) free induction decay from the first pulse, the diffusion factor

$$E(\cos\phi_{\tau_1,\,o})E(\cos\phi_{2\tau_1,\,\tau_1})E(\cos(\eta_{\tau_1,\,o}\cdot\tau_1) = \exp\left[-\frac{1}{3}k\tau_1^3(1+1+3)\right] = \exp(-\frac{5}{3}k\tau_1^3) \tag{11}$$

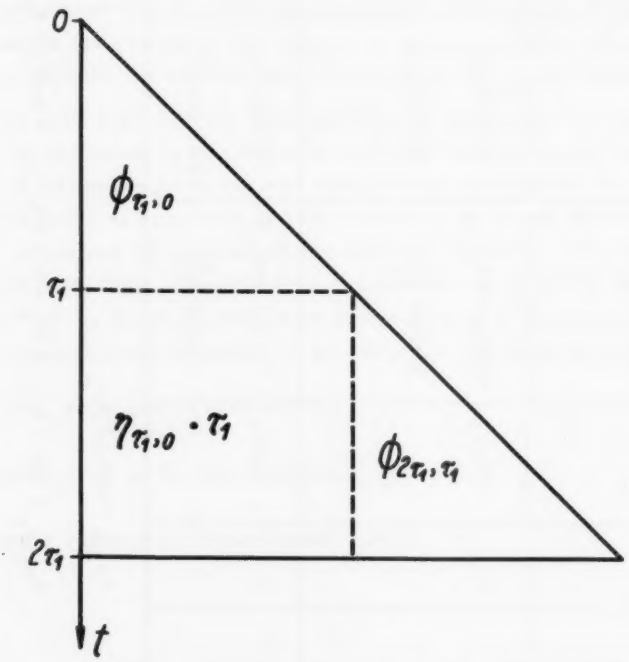

Fig. 3. Component-sums of $\phi_{t,\,o}$ after Das and Saha.

Considering the array (6) for $\phi_{t,o}$: $\phi_{2\tau_1,o} = \phi_{\tau_1,o} + \phi_{2\tau_1,\tau_1} + \eta_{\tau_1,o} \cdot \tau_1$ (see fig. 3) and from (3) it then immediately follows that Das and Saha would arrive at exactly the same result as Hahn if $\phi_{\tau_1,o}$, $\phi_{2\tau_1,\tau_1}$ and $\eta_{\tau_1,o}$ were mutually independent, that is if the condition for their calculations did hold. Since in the array for $\phi_{t,o}$ the same η is repeated in vertical direction while η's in a horizontal line are mutually independent $\phi_{2\tau_1,\tau_1}$ is of course independent of both $\phi_{\tau_1,o}$ and $\eta_{\tau_1,o}$; however, $\phi_{\tau_1,o}$ and $\eta_{\tau_1,o}$ are directly correlated. (In general, any two η or ϕ are always correlated when they belong to time intervals overlapping each other; otherwise always uncorrelated. In other words the triangle for $\phi_{t,o}$ may be divided into mutually independent component-sums by vertical, and only by vertical, lines).

Corrected calculations

In the following will be given the revised theory taking account of the possibility of correlation between component-sums of $\phi_{t,o}$. From table I the signal V(E) for $\tau_1 < t < \tau_2 > 2\tau_1$, replacing τ_2 with t, is found to be:

$$\sin\xi_1 \exp(-\frac{t}{T_2})E \begin{bmatrix} \sin A_{\tau_1,o} \sin A_{t,\tau_1} \\ -\cos\xi_2 \cos A_{\tau_1,o} \cos A_{t,\tau_1} \end{bmatrix}$$

$$-W(\tau_1) \sin\xi_2 \exp(-\frac{t-\tau_1}{T_2}) E(\cos A_{t,\tau_1})$$

The bracket from the first term gives

$$E \begin{bmatrix} (\sin^2\frac{\xi_2}{2} + \cos^2\frac{\xi_2}{2}) \sin A_{\tau_1,o} \sin A_{t,\tau_1} \\ +(\sin^2\frac{\xi_2}{2} - \cos^2\frac{\xi_2}{2}) \cos A_{\tau_1,o} \cos A_{t,\tau_1} \end{bmatrix} =$$

$$E\left[-\cos^2\frac{\xi_2}{2} \cos(A_{t,\tau_1} + A_{\tau_1,o}) + \sin^2\frac{\xi_2}{2} \cos(A_{t,\tau_1} - A_{\tau_1,o})\right]$$

Thus V(E) is:

$$-\sin\xi_1\cos^2\frac{\xi_2}{2}\exp(-\frac{t}{T_2})E\left[\cos(\Delta\omega t+\phi_{t,o})\right]$$

$$+\sin\xi_1\sin^2\frac{\xi_2}{2}\exp(-\frac{t}{T_2})E\left[\cos\left\{\Delta\omega(t-2\tau_1)+(\phi_{t,o}-2\phi_{\tau_1,o})\right\}\right]$$

$$-W(\tau_1)\sin\xi_2\exp(-\frac{t-\tau_1}{T_2})E\left[\cos\left\{\Delta\omega(t-\tau_1)+(\phi_{t,o}-\phi_{\tau_1,o})\right\}\right]$$

or, by using (3) and regarding fig. 4 indicating how the resulting diffusion sum ϕ_{res} (enclosed in the heavy lines) may be divided into mutually independent component-sums

V(E) =

$$-\sin\xi_1\cos^2\frac{\xi_2}{2}\exp(-\frac{t}{T_2})\exp(-\frac{t^2}{2T_3^3})\exp(-\frac{1}{3}kt^3) \qquad (12a)$$

$$+\sin\xi_1\sin^2\frac{\xi_2}{2}\exp(-\frac{t}{T_2})\exp\left[-\frac{(t-2\tau_1)^2}{2T_3^2}\right]\exp\left[-\frac{k}{3}\left\{2(t-\tau_1)^3-(t-2\tau_1)^3\right\}\right] \qquad (12b)$$

$$-W(\tau_1)\sin\xi_2\exp(-\frac{t-\tau_1}{T_2})\exp\left[-\frac{(t-\tau_1)^2}{2T_3^2}\right]\exp\left[-\frac{k}{3}\left\{3\tau_1(t-\tau_1)^2+(t-\tau_1)^3\right\}\right] \qquad (12c)$$

(12a) is the fraction $\cos^2\frac{\xi_2}{2}$ of the free induction from the first pulse which continues with unaltered diffusion term.

(12b) is the primary echo formed by the second pulse from the rest of the decay from the first pulse. The echo has reduced diffusion damping, and in the special case of echo maximum $t=2\tau_1$ the factor is $\exp(-\frac{2}{3}k\tau_1^3)$ in agreement with Carr and Purcell and with Torrey[4].

(12c) is the free-induction decay following the second pulse.

As the normal spin-echo conditions are of course assumed to be satisfied, inserting $\xi_2 = 360^\circ$ is equivalent to omitting the second pulse, and as one would expect, the total free induction decay from the first pulse is seen to continue unchanged, while the echo with reduced diffusion damping does not occur. Das and Saha, however, obtain for both

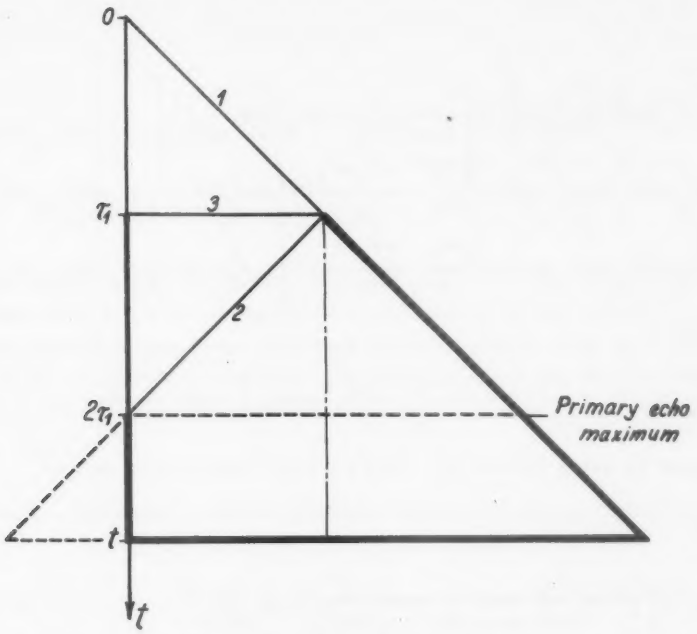

Fig. 4. Illustration of ϕ_{res} for the three components of the signal following the second pulse.

the echo and the modified decay the same reduction of diffusion damping (see (11)) which does not seem reasonable.

Performing quite similar calculations V(H), $\tau_2 < t < \tau_3 > 2\tau_2$, is found to be:

$$-\sin\xi_1\cos^2\frac{\xi_2}{2}\cos^2\frac{\xi_3}{2}\exp(-\frac{t}{T_2})E\left[\cos(\Delta\omega t + \phi_{t,o})\right]$$

$$+\sin\xi_1\sin^2\frac{\xi_2}{2}\cos^2\frac{\xi_3}{2}\exp(-\frac{t}{T_2})E\left[\cos\left\{\Delta\omega(t-2\tau_1) + (\phi_{t,o}-2\phi_{\tau_1,o})\right\}\right]$$

$$+\sin\xi_1\cos^2\frac{\xi_2}{2}\sin^2\frac{\xi_3}{2}\exp(-\frac{t}{T_2})E\left[\cos\left\{\Delta\omega(t-2\tau_2) + (\phi_{t,o}-2\phi_{\tau_2,o})\right\}\right]$$

$$-\sin\xi_1\sin^2\frac{\xi_2}{2}\sin^2\frac{\xi_3}{2}\exp(-\frac{t}{T_2})E\left[\cos\left\{\Delta\omega(t-2\tau_2+2\tau_1) + (\phi_{t,o}-2\phi_{\tau_2,o}+2\phi_{\tau_1,o})\right\}\right]$$

$$-W(\tau_1)\sin\xi_2\cos^2\frac{\xi_3}{2}\exp(-\frac{t-\tau_1}{T_2})E\left[\cos\left\{\Delta\omega(t-\tau_1) + (\phi_{t,o}-\phi_{\tau_1,o})\right\}\right]$$

$$+W(\tau_1)\sin\xi_2\sin^2\frac{\xi_3}{2}\exp(-\frac{t-\tau_1}{T_2})E\left[\cos\left\{\Delta\omega(t-2\tau_2+\tau_1) + (\phi_{t,o}-2\phi_{\tau_2,o}+\phi_{\tau_1,o})\right\}\right]$$

$$-K_2 \sin\xi_3 \exp\left(-\frac{t-\tau_2}{T_2}\right) E\left[\cos\left\{\Delta\omega(t-\tau_2) + (\phi_{t,o}-\phi_{\tau_2,o})\right\}\right]$$

$$+\tfrac{1}{2}\sin\xi_1 \sin\xi_2 \sin\xi_3 \exp\left(-\frac{\tau_2-\tau_1}{T_1} - \frac{t-\tau_2+\tau_1}{T_2}\right) E\left[\cos\left\{\Delta\omega(t-\tau_2+\tau_1)+(\phi_{t,o}-\phi_{\tau_2,o}+\phi_{\tau_1,o})\right\}\right]$$

$$+\tfrac{1}{2}\sin\xi_1 \sin\xi_2 \sin\xi_3 \exp\left(-\frac{\tau_2-\tau_1}{T_1} - \frac{t-\tau_2+\tau_1}{T_2}\right) E\left[\cos\left\{\Delta\omega(t-\tau_2-\tau_1)+(\phi_{t,o}-\phi_{\tau_2,o}-\phi_{\tau_1,o})\right\}\right]$$

where

$$K_2 = K_2(\xi_2, \xi_1, \tau_2, \tau_1, T_1) = 1 - \left[1 - W(\tau_1)\cos\xi_2\right] \exp\left(-\frac{\tau_2-\tau_1}{T_1}\right)$$

Again by using (3) and fig. 5 for j = 3 (an extension of fig. 4)

$$V(H) =$$

$$-\sin\xi_1 \cos^2\frac{\xi_2}{2} \cos^2\frac{\xi_2}{2} \exp\left(-\frac{t}{T_2}\right) \exp\left(-\frac{t^2}{2T_3^2}\right) \exp\left(-\tfrac{1}{3}kt^3\right) \qquad (13a)$$

$$+\sin\xi_1 \sin^2\frac{\xi_2}{2} \cos^2\frac{\xi_3}{2} \exp\left(-\frac{t}{T_2}\right) \exp\left[-\frac{(t-2\tau_1)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{2(t-\tau_1)^3-(t-2\tau_1)^3\right\}\right] \qquad (13b)$$

$$+\sin\xi_1 \cos^2\frac{\xi_2}{2} \sin^2\frac{\xi_3}{2} \exp\left(-\frac{t}{T_2}\right) \exp\left[-\frac{(t-2\tau_2)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{2(t-\tau_2)^3-(t-2\tau_2)^3\right\}\right] \qquad (13c)$$

$$-\sin\xi_1 \sin^2\frac{\xi_2}{2} \sin^2\frac{\xi_3}{2} \exp\left(-\frac{t}{T_2}\right) \exp\left[-\frac{(t-2\tau_2+2\tau_1)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{2(t-\tau_2)^3-2(t-2\tau_2+\tau_1)^3+\right.\right.$$
$$\left.\left.+(t-2\tau_2+2\tau_1)^3\right\}\right] \qquad (13d)$$

$$-W(\tau_1)\sin\xi_2 \cos^2\frac{\xi_3}{2} \exp\left(-\frac{t-\tau_1}{T_2}\right) \exp\left[-\frac{(t-\tau_1)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{(t-\tau_1)^3+3\tau_1(t-\tau_1)^2\right\}\right] \qquad (13e)$$

$$+W(\tau_1)\sin\xi_2 \sin^2\frac{\xi_3}{2} \exp\left(-\frac{t-\tau_1}{T_2}\right) \exp\left[-\frac{(t-2\tau_2+\tau_1)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{2(t-\tau_2)^3-(t-2\tau_2+\tau_1)^3+\right.\right.$$
$$\left.\left.+3\tau_1(t-2\tau_1+\tau_1)^2\right\}\right] \qquad (13f)$$

$$-K_2 \sin\xi_3 \exp\left(-\frac{t-\tau_2}{T_2}\right) \exp\left[-\frac{(t-\tau_2)^2}{2T_3^2}\right] \exp\left[-\frac{k}{3}\left\{(t-\tau_2)^3+3\tau_2(t-\tau_2)^2\right\}\right] \qquad (13g)$$

$$+\tfrac{1}{2}\sin\xi_1 \sin\xi_2 \sin\xi_3 \exp\left(-\frac{\tau_2-\tau_1}{T_1} - \frac{t-\tau_2+\tau_1}{T_2}\right) \exp\left[-\frac{(t-\tau_2+\tau_1)^2}{2T_3^2}\right]$$
$$\times \exp\left[-\frac{k}{3}\left\{3(t-\tau_2)^2(\tau_2-\tau_1) + (t-\tau_2+\tau_1)^3\right\}\right] \qquad (13h)$$

$$+\tfrac{1}{2}\sin\xi_1\sin\xi_2\sin\xi_3\exp\left(-\frac{\tau_2-\tau_1}{T_1}-\frac{t-\tau_2+\tau_1}{T_2}\right)\exp\left[-\frac{(t-\tau_2-\tau_1)^2}{2T_3^2}\right]$$

$$\times\exp\left[-\tfrac{k}{3}\left\{2(t-\tau_2)^3+3(t-\tau_2)^2(\tau_2-\tau_1)-(t-\tau_2-\tau_1)^3\right\}\right] \qquad (13i)$$

Here (13a, b, e) are modifications of (12a, b, c) with unaltered diffusion terms, (13c, d, f) are echoes of (12a, b, c) caused by the third pulse, (13g) is the free-induction decay following the third pulse, (13h) is a virtual and (13i) a real stimulated echo. Table II gives for the four secondary echoes the diffusion terms at echo maximum (compare table III, ref. 2, pag. 754).

Table II. Corrected values of diffusion terms for the secondary echoes.

Term	Echo maximum	Value of diffusion term at echo maximum	
Stimulated echo	$\tau_2+\tau_1$	$\exp\left[-\tfrac{k}{3}(3\tau_1^2\tau_2-\tau_1^3)\right]$	(13i)
First secondary echo	$2\tau_2-2\tau_1$	$\exp\left[-\tfrac{2}{3}k\left\{\tau_1^3+(\tau_2-2\tau_1)^3\right\}\right]$	(13d)
Second secondary echo	$2\tau_2-\tau_1$	$\exp\left[-\tfrac{2}{3}k(\tau_2-\tau_1)^3\right]$	(13f)
Third secondary echo	$2\tau_2$	$\exp\left[-\tfrac{2}{3}k\tau_2^3\right]$	(13c)

It is now obvious how to continue with four pulses or more by extending table I. On account of the striking systematic nature of the results, however, it will be convenient to proceed directly to the following recursion formula, obtained simply by analysing the "transport" of terms in the extended table I. For $\tau_{n-1} < t < \tau_n > 2\tau_{n-1}$,

$$V_n(t) = \sum_{i=1}^{x(n)} B_{i,n}(t)\, E\left[\cos\left\{\Delta\omega(t-t_{i,n})+(\phi_{t,o}-\phi_{i,n})\right\}\right] =$$

$$\sum_{i=1}^{x(n-1)} B_{i,n-1}\exp\left(-\frac{t-\tau_{n-1}}{T_2}\right)\left[\cos^2\frac{\xi_n}{2}\, E\left[\cos\left\{\Delta\omega(t-t_{i,n-1})+(\phi_{t,o}-\phi_{i,n-1})\right\}\right]\right.$$

$$\left. -\sin^2\frac{\xi_n}{2}\, E\left[\cos\left\{\Delta\omega(t-2\tau_{n-1}+t_{i,n-1})+(\phi_{t,o}-2\phi_{\tau_{n-1},o}+\phi_{i,n-1})\right\}\right]\right]$$

$$+\sum_{i=1}^{x(n-2)}\frac{1}{2}B_{i,n-2}\sin\xi_{n-1}\sin\xi_n\exp\left(-\frac{\tau_{n-1}-\tau_{n-2}}{T_1}-\frac{t-\tau_{n-1}}{T_2}\right)$$

$$\times\left[E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}+\tau_{n-2}-t_{i,n-2})+(\phi_{t,o}-\phi_{\tau_{n-1},o}+\phi_{\tau_{n-2},o}-\phi_{i,n-2})\right\}\right]\right.$$

$$\left. +\, E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}-\tau_{n-2}+t_{i,n-2})+(\phi_{t,o}-\phi_{\tau_{n-1},o}-\phi_{\tau_{n-2},o}+\phi_{i,n-2})\right\}\right]\right]$$

$$+\sum_{i=1}^{x(n-3)}\frac{1}{2}B_{i,n-3}\sin\xi_{n-2}\cos\xi_{n-1}\sin\xi_n\exp\left(-\frac{\tau_{n-1}-\tau_{n-3}}{T_1}-\frac{t-\tau_{n-1}}{T_2}\right)$$

$$\times\left[E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}+\tau_{n-3}-t_{i,n-3})+(\phi_{t,o}-\phi_{\tau_{n-1},o}+\phi_{\tau_{n-3},o}-\phi_{i,n-3})\right\}\right]\right.$$

$$\left. +E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}-\tau_{n-3}+t_{i,n-3})+(\phi_{t,o}-\phi_{\tau_{n-1},o}-\phi_{\tau_{n-3},o}+\phi_{i,n-3})\right\}\right]\right]$$

$$+ \ldots\ldots$$

$$+\sum_{i=1}^{x(j)}\frac{1}{2}B_{i,j}\sin\xi_{j+1}\cos\xi_{j+2}\ldots\cos\xi_{n-1}\sin\xi_n\exp\left(-\frac{\tau_{n-1}-\tau_j}{T_1}-\frac{t-\tau_{n-1}}{T_2}\right)$$

$$\times\left[E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}+\tau_j-t_{i,j})+(\phi_{t,o}-\phi_{\tau_{n-1},o}+\phi_{\tau_j,o}-\phi_{i,j})\right\}\right]\right.$$

$$\left. +E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}-\tau_j+t_{i,j})+(\phi_{t,o}-\phi_{\tau_{n-1},o}-\phi_{\tau_j,o}+\phi_{i,j})\right\}\right]\right]$$

$$+ \ldots\ldots$$

$$+\sum_{i=1}^{x(1)} \frac{1}{2} B_{i,1} \sin\xi_2 \cos\xi_3 \cdots \cos\xi_{n-1} \sin\xi_n \exp\left(-\frac{\tau_{n-1}-\tau_1}{T_1} - \frac{t-\tau_{n-1}}{T_2}\right)$$

$$\times \left[E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}+\tau_1-t_{i,1})+(\phi_{t,o}-\phi_{\tau_{n-1},o}+\phi_{\tau_1,o}-\phi_{i,1})\right\}\right]\right.$$

$$\left. +E\left[\cos\left\{\Delta\omega(t-\tau_{n-1}-\tau_1+t_{i,1})+(\phi_{t,o}-\phi_{\tau_{n-1},o}-\phi_{\tau_1,o}+\phi_{i,1})\right\}\right]\right]$$

$$- K_{n-1} \sin\xi_n \exp\left(-\frac{t-\tau_{n-1}}{T_2}\right) E\left[\cos\left\{\Delta\omega(t-\tau_{n-1})+(\phi_{t,o}-\phi_{\tau_{n-1},o})\right\}\right] \qquad (14)$$

where

$K_n = K_n(\xi_n, \xi_{n-1}, \ldots \xi_1, \tau_n, \tau_{n-1}, \ldots \tau_1, T_1)$, as

$K_o = W(0) = 1$

$K_1 = W(\tau_1) = 1 - (1-\cos\xi_1)\exp\left(-\frac{\tau_1}{T_1}\right)$

$K_2 = 1 - (1-K_1 \cdot \cos\xi_2)\exp\left(-\frac{\tau_2-\tau_1}{T_1}\right)$

. . . . .

$$K_n = 1 - (1-K_{n-1}\cos\xi_n)\exp\left(-\frac{\tau_n-\tau_{n-1}}{T_1}\right) \qquad (15)$$

and where $B_{i,j} = B_{i,j}(\tau_j)$, starting with:

$$B_{i,1} = B_{1,1} = B_{1,1}(\tau_1) = -K_o \sin\xi_1 \exp\left(-\frac{\tau_1}{T_2}\right) = -\sin\xi_1 \exp\left(-\frac{\tau_1}{T_2}\right)$$

The number x(j) of signal components for $\tau_{j-1} < t < \tau_1 > 2\tau_{j-1} (j=1, 2, 3\ldots)$ is seen to start with: x(1) = 1 ; x(2) = 3 ; etc. Likewise,

$$\phi_{i,1} = 0 \quad , \quad \phi_{i,2} = \begin{cases} 0 \\ \phi_{\tau_1,o} \\ 2\phi_{\tau_1,o} \end{cases} \quad \text{etc.}$$

$$t_{i,1} = 0 \quad , \quad t_{i,2} = \begin{cases} 0 \\ \tau_1 \\ 2\tau_1 \end{cases} \quad \text{etc.}$$

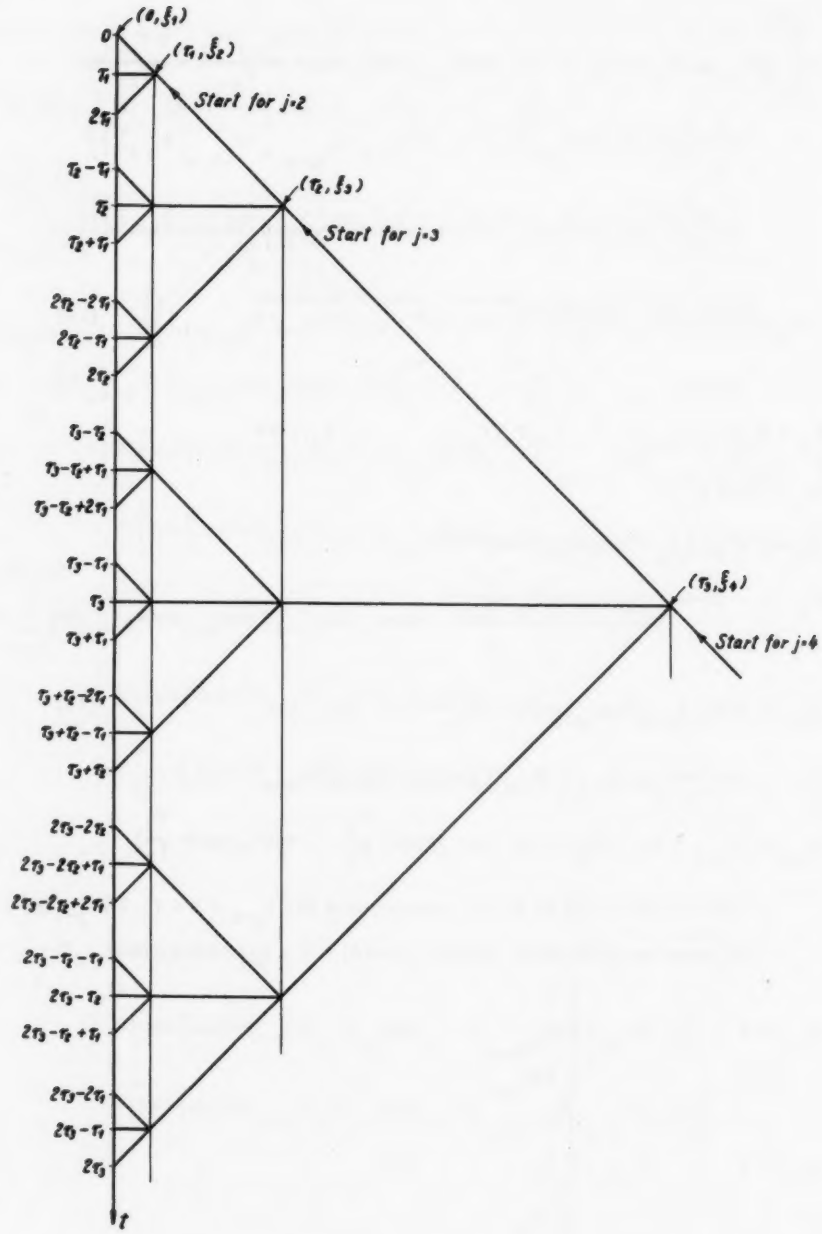

Fig. 5. "Spin-echo diagram" (for four pulses or less).

(14) leads to the simple "spin-echo-diagram" fig. 5 from which any information about any signal component for a wanted number of pulses may be obtained according to the following directions. Starting as indicated for a wanted pulse number j and choosing, when crossing each of the vertical lines corresponding to pulses, one of the three possibilities, (the horizontal or one of the two inclined lines), the "maximum times" $t_{i,j}$ for the number x(j) of signal terms $\tau_{j-1} < t < \tau_j$ are simply found as the points where the system of broken lines crosses the time axis (which is the vertical line corresponding to the first pulse). Thus the total number of component terms due to n pulses is:

$$M(n) = \sum_{j=1}^{n} x(j) = \sum_{j=1}^{n} 3^{j-1} = \tfrac{1}{2}(3^n-1)$$

However, x(j) is made up of real echoes, virtual echoes, and of pulse decays. If $\tau_j > 2\tau_{j-1}$, as assumed for simplicity in the previous calculations and for fig. 5, the total number of real echoes is:

$$N(n) = \sum_{j=2}^{n} y(j) = \sum_{j=2}^{n} \sum_{k=2}^{j} 3^{k-2} = \sum_{j=2}^{n} \tfrac{1}{2}(3^{j-1}-1) = \tfrac{1}{4}(3^n-2n-1) \qquad (17)$$

which result has been derived before (see ref. 5).

The trigonometrical coefficients of the different signal terms are also obtainable from the diagram, observing the following rules: Immediately after "start" in the diagram, and also whenever moving along an inclined line, then upon crossing the j'th vertical pulse line,

a) if continuing in the same direction: multiply by $\cos^2 \frac{\xi_j}{2}$

b) if choosing the other inclined line: multiply by $-\sin^2 \frac{\xi_j}{2}$

c) if choosing the horizontal line: multiply by $-\sin\xi_j$.

When coming from a horizontal line and crossing the j'th vertical pulse line,

a) if continuing in the same direction: multiply by $\cos\xi_j$

b) if choosing one of the inclined lines: multiply by $\frac{1}{2}\sin\xi_j$.

However, if after crossing the j'th vertical pulse line the path continues in a horizontal direction, it is necessary to insert the factor $-K_{j-1}\sin\xi_j$. Especially by intersecting the time axis the factor $-K_o\sin\xi_1 = -\sin\xi_1$ should be inserted. The K's are the only factors of the expressions for the various signal components which are combined trigonometrical and (longitudinal) relaxational terms (see (15)).

Pure exponential transversal relaxation damping is present during the total time corresponding to inclined lines. Pure exponential longitudinal relaxation damping is present during the total time corresponding to horizontal lines, (excepting a horizontal "K-line" touching the time axis, for which as mentioned the longitudinal damping is not simply exponential).

Only the diffusion damping remains to be considered. The factor for this is seen to be $\exp\left\{-\frac{1}{2}\,E\left[\phi_{i,j\ res}(t)\right]^2\right\}$, where $\phi_{i,j\ res}(t) = \phi_{t,o} - \phi_{i,j}$ is the resulting random diffusion sum enclosed between the horizontal line which corresponds to the wanted time t, and the broken line which corresponds to the considered i'th number of signal components following the j'th pulse. The geometrical figure containing ϕ_{res} is divided by vertical lines into rectangles, triangles, and trapezoids containing mutually independent normally distributed diffusion sums. As the variance of the sum within a trapezoid is the difference between the variances of the sums within two triangles, the diffusion factor is easily obtained by using (3) together with (5) and (7).

As a correction of previous results obtained by Das and Roy (see ref. 6, table I, pag. 526) the signal components following the fourth pulse, $\tau_3 < t < \tau_4 > 2\tau_3$, will now be taken directly from the spin-echo diagram fig. 5 (The more or less modified decays from previous pulses and echoes are omitted as they are of very little interest).

$$V(t) =$$

$$+\sin\xi_1\cos^2\frac{\xi_2}{2}\cos^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t}{T_2})\exp\left[-\frac{(t-2\tau_3)^2}{2T_3^2}\right]\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-(t-2\tau_3)^3\right\}\right]$$

$$-\sin\xi_1\sin^2\frac{\xi_2}{2}\cos^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t}{T_2})\exp\left[-\frac{(t-2\tau_3+2\tau_1)^2}{2T_3^2}\right]$$
$$\times\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-2(t-2\tau_3+\tau_1)^3+(t-2\tau_3+2\tau_1)^3\right\}\right]$$

$$-\sin\xi_1\cos^2\frac{\xi_2}{2}\sin^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t}{T_2})\exp\left[-\frac{(t-2\tau_3+2\tau_2)^2}{2T_3^2}\right]$$
$$\times\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-2(t-2\tau_3+\tau_2)^3+(t-2\tau_3+2\tau_2)^3\right\}\right]$$

$$+\sin\xi_1\sin^2\frac{\xi_2}{2}\sin^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t}{T_2})\exp\left[-\frac{(t-2\tau_3+2\tau_2-2\tau_1)^2}{2T_3^2}\right]$$
$$\times\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-2(t-2\tau_3+\tau_2)^3+2(t-2\tau_3+2\tau_2-\tau_1)^3-(t-2\tau_3+2\tau_2-2\tau_1)^3\right\}\right]$$

$$+W(\tau_1)\sin\xi_2\cos^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t-\tau_1}{T_2})\exp\left[-\frac{(t-2\tau_3+\tau_1)^2}{2T_3^2}\right]$$
$$\times\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-(t-2\tau_3+\tau_1)^3+3\tau_1(t-2\tau_3+\tau_1)^2\right\}\right]$$

$$-W(\tau_1)\sin\xi_2\sin^2\frac{\xi_3}{2}\sin^2\frac{\xi_4}{2}\exp(-\frac{t-\tau_1}{T_2})\exp\left[-\frac{(t-2\tau_3+2\tau_2-\tau_1)^2}{2T_3^2}\right]$$
$$\times\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-2(t-2\tau_3+\tau_2)^3+(t-2\tau_3+2\tau_2-\tau_1)^3+3\tau_1(t-2\tau_3+2\tau_2-\tau_1)^2\right\}\right]$$

$$+K_2\sin\xi_3\sin^2\frac{\xi_4}{2}\exp(-\frac{t-\tau_2}{T_2})\exp\left[-\frac{(t-2\tau_3+\tau_2)^2}{2T_3^2}\right]\exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-(t-2\tau_3+\tau_2)^3+\right.\right.$$
$$\left.\left.+3\tau_2(t-2\tau_3+\tau_2)^2\right\}\right]$$

$$-\frac{1}{2}\sin\xi_1\sin\xi_2\sin\xi_3\sin^2\frac{\xi_4}{2}\exp(-\frac{\tau_2-\tau_1}{T_1}-\frac{t-\tau_2+\tau_1}{T_2})\exp\left[-\frac{(t-2\tau_3+\tau_2-\tau_1)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3+3(t-2\tau_3+\tau_2)^2(\tau_2-\tau_1)-(t-2\tau_3+\tau_2-\tau_1)^3\right\}\right]$$

$$-\frac{1}{2}\sin\xi_1\sin\xi_2\sin\xi_3\sin^2\frac{\xi_4}{2}\exp\left(-\frac{\tau_2-\tau_1}{T_1}-\frac{t-\tau_2+\tau_1}{T_2}\right)\exp\left[-\frac{(t-2\tau_3+\tau_2+\tau_1)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3-2(t-2\tau_3+\tau_2)^3+3(t-2\tau_3+\tau_2)^2(\tau_2-\tau_1)+(t-2\tau_3+\tau_2-\tau_1)^3\right\}\right]$$

$$+\sin\xi_1\cos^2\frac{\xi_2}{2}\sin\xi_3\sin^2\frac{\xi_4}{2}\exp\left(-\frac{\tau_3-\tau_2}{T_1}-\frac{t-\tau_3+\tau_2}{T_2}\right)\exp\left[-\frac{(t-\tau_3-\tau_2)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3+3(t-\tau_3)^2(\tau_3-\tau_2)-(t-\tau_3-\tau_2)^3\right\}\right]$$

$$-\sin\xi_1\sin^2\frac{\xi_2}{2}\sin\xi_3\sin^2\frac{\xi_4}{2}\exp\left(-\frac{\tau_3-\tau_2}{T_1}-\frac{t-\tau_3+\tau_2}{T_2}\right)\exp\left[-\frac{(t-\tau_3-\tau_2+2\tau_1)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3+3(t-\tau_3)^2(\tau_3-\tau_2)-2(t-\tau_3-\tau_2+\tau_1)^3+(t-\tau_3-\tau_2+2\tau_1)^3\right\}\right]$$

$$+W(\tau_1)\sin\xi_2\sin\xi_3\sin^2\frac{\xi_4}{2}\exp\left(-\frac{\tau_3-\tau_2}{T_1}-\frac{t-\tau_3+\tau_2-\tau_1}{T_2}\right)\exp\left[-\frac{(t-\tau_3-\tau_2+\tau_1)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3+3(t-\tau_3)^2(\tau_3-\tau_2)-(t-\tau_3-\tau_2+\tau_1)^3+3\tau_1(t-\tau_3-\tau_2+\tau_1)^2\right\}\right]$$

$$+\frac{1}{2}\sin\xi_1\sin\xi_2\cos\xi_3\sin\xi_4\exp\left(-\frac{\tau_3-\tau_1}{T_1}-\frac{t-\tau_3+\tau_1}{T_2}\right)\exp\left[-\frac{(t-\tau_3-\tau_1)^2}{2T_3^2}\right]$$

$$\times \exp\left[-\frac{k}{3}\left\{2(t-\tau_3)^3+3(t-\tau_3)^2(\tau_3-\tau_1)-(t-\tau_3-\tau_1)^3\right\}\right]$$

$$-K_3\sin\xi_4\exp\left(-\frac{t-\tau_3}{T_2}\right)\exp\left[-\frac{(t-\tau_3)^2}{2T_3^2}\right]\exp\left[-\frac{k}{3}\left\{3\tau_3(t-\tau_3)^2+(t-\tau_3)^3\right\}\right] \qquad (18)$$

$$K_3 = 1-\left[1-\left[1-\left\{1-W(\tau_1)\cos\xi_2\right\}\exp\left(-\frac{\tau_2-\tau_1}{T_1}\right)\right]\cos\xi_3\right]\exp\left(-\frac{\tau_3-\tau_2}{T_1}\right) \qquad (19)$$

So far it has been assumed that $\tau_j > 2\tau_{j-1}$, but from the very way in which the calculations are performed it follows that $t_{i,j}$ and $\phi_{i,j\ res}(t)$ will be given by the same expressions, whichever pulse train is used. So also a spin-echo diagram like fig. 5 remains valid for any combinations of pulses, and the expressions for the various terms can still be derived from the diagram, following the above given directions. The only difference resulting from supplying new pulses before all the echoes from the previous one have formed is that the number N(n) of real echoes is reduced (as some of the otherwise real echoes now become virtual), and that the echoes formed become further (triginometrically) modified.

For an echo which is the result of repeated echo formations it will be seen that the diffusion damping at echo maximum is the accumulation of the diffusion dampings for each echo formation. This follows simply from the fact that ϕ_{res} is enclosed between the broken line for the considered echo and the horizontal time-line at echo maximum, and echoes occur when those two lines are crossing each other.

For the Carr-Purcell pulse train ($\xi_1=90^o$, $\xi_j=180^o$, j=2, 3,.., n) the trigonometrical part of the coefficient is:

$$-\sin\xi_1(-1)^{j-1}\prod_{i=2}^{j}\sin^2\frac{\xi_i}{2} = (-1)^j.$$

Theoretically it is possible to adjust to this value if the pulse train is assumed to be stationary, and if the radiofrequency field is sufficiently homogeneous. If, however, inhomogenity is present the ideal rotation angle cannot be obtained for all of the spins, and a damping (besides possible adjustment damping) will occur, the factor being:

$$1-E(\Delta\xi_1)^2 - \frac{1}{2}\sum_{i=2}^{j}E(\Delta\xi_i)^2 \simeq 1-\frac{j}{2}E(\Delta\xi)^2\ ,\ \text{for } j\gg 1 \text{ and } \Delta\xi_i = \Delta\xi.$$

By means of a spin-echo diagram, however, it is easy to see that with the ratios of pulse intervals, 1:2:2:2:2 etc. used by Carr and Purcell a lot of disturbing echoes will occur exactly at the maximum times for the considered echo array. Among the disturbing echoes one group is quite dominating, namely stimulated echoes (and repetitions of same) caused by the pulse train together with the first pulse and the considered echo array. They will all have the size $\simeq \frac{1}{2}E(\Delta\xi)^2$, and will all have

the same sign which is opposite to but shifting in accordance to that of an echo of the considered array. Since the number of disturbing echoes is $\frac{1}{2}(j-1)(j-2) \approx \frac{1}{2}j^2$ for $j \gg 1$ they will give a damping factor: $1-\frac{1}{2}j^2 E(\Delta\xi)^2$. Thus the fault due to disturbing echoes largely exceeds the fault on the considered array itself. (Since $T_1 \geq T_2$ this effect is normally increased by taking account of relaxation). It may be of some interest to note that for the same pulse train modified to have the pulse interval ratios 2:6:7:7:7 etc. all disturbing echoes from a spin-echo diagram are seen to be separated from the considered array. Furthermore their size may be used to some extent to indicate whether the pulse angles have the ideal values or not.

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